Equations with random coefficients and theories of random fluctuations

Problems with small scale structures abound in applied sciences. Their detailed microscopic description is often not available or generates computationally intractable problems. Homogenization theory has then been developed to understand the influence of micro-structures at a macroscopic level. The assumptions on the micro-structure under which homogenization holds, such as, e.g., periodicit or ergodicity, are often not satisfied in practice. However, the homogenization point of view proves to be very fruitful in, e.g., macroscopic parameter estimations, and to assess how multi-scale algorithms fare in well-controlled settings.

Equally important in practice, but often much difficult to study, is the analysis of random fluctuations beyond the homogenization limit. These fluctuations model noise in parameter estimation measurements and limit reconstruction resolution. This talk reviews some recent results obtained on the random fluctuations of solutions of partial differential equations with random coefficients. We then analyze conditions under which multiscale algorithms correctly capture such random fluctuations.